# **GATE**2024

# **Civil Engineering**



- ✓ Fully solved with explanations
- Analysis of previous papers
- ▼ Topicwise presentation
- ▼ Thoroughly revised & updated





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#### GATE - 2024 Civil Engineering

Topicwise Previous GATE Solved Papers (1994-2023)

#### Editions

1st Edition : 2008 2<sup>nd</sup> Edition : 2009 3<sup>rd</sup> Edition : 2010 4<sup>th</sup> Edition : 2011 5<sup>th</sup> Edition : 2012 6<sup>th</sup> Edition : 2013 7<sup>th</sup> Edition : 2014 8<sup>th</sup> Edition : 2015 9<sup>th</sup> Edition : 2016 10<sup>th</sup> Edition : 2017 11th Edition : 2018 12<sup>th</sup> Edition : 2019 13th Edition : 2020 14<sup>th</sup> Edition : 2021 15<sup>th</sup> Edition : 2022

: 2023

16th Edition

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# Preface

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

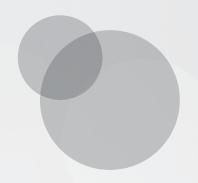


The new edition of **GATE 2024 Solved Papers : Civil Engineering** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group



# **GATE-2024**

## **Civil Engineering**

## **CONTENTS**

1.	Solid Mechanics	1-65
2.	Structural Analysis	66-126
3.	RCC Structures and Prestressed Concrete	127-168
4.	Design of Steel Structures	169-210
5.	Geotechnical Engineering	211-319
6.	Fluid Mechanics & Fluid Machines	320-402
7.	Environmental Engineering	403-483
8.	Irrigation Engineering	484-504
9.	Engineering Hydrology	505-535
10.	Transportation Engineering	536-601
11.	Geometics Engineering	602-630
12.	Construction Materials, Construction Managem	ent
	and Engg. Mechanics	631-649
13.	Engineering Mathematics	650-738
14.	General Aptitude	739-791

### **Solid Mechanics**

# UNIT

# CONTENTS

1.	Properties of Metals, Stress & Strain	3
		_

- 2. Shear Force and Bending Moment 15
- 3. Principal Stress and Principal Strain 24
- 4. Bending and Shear Stresses 31
- 5. Deflection of BeaM 38
- 6. Torsion of Shafts and Pressure Vessels 55
- 7. Theory of Columns & Shear Centre 60

# Solid Mechanics

# Syllabus

Bending moment and shear force in statically determinate beaM; Simple stress and strain relationships; Simple bending theory, flexural and shear stresses, shear centre; Uniform torsion, Transformation of stress; buckling of column, combined and direct bending stresses.

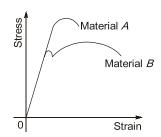
#### **Analysis of Previous GATE Papers**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks				
1994	2	2	6				
1995	2	_	2				
1996	3	_	3				
1997	2	_	2				
1998	_	1	2				
1999	4	_	4				
2000	7	_	7				
2001	1	_	1				
2002	2	1	4				
<b>200</b> 3	2	5	12				
2004	1	6	13				
2005	2	3	8				
2006	3	9	21				
2007	3	5	13				
2008	_	8	16				
2009	2	5	12				
2010	5	1	7				
2011	1	3	7				
2012	4	2	8				
2013	4	2 8					

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks		
2014 Set-1	1	3	7		
2014 Set-2	1	5	11		
2015 Set-1	1	2	5		
2015 Set-2	2	3	8		
2016 Set-1	_	3	6		
2016 Set-2	-	3	6		
2017 Set-1	2	2	6		
2017 Set 2	1	4	9		
2018 Set-1	2	3	8		
2018 Set-2	_	1	2		
2019 Set-1	2	1	4		
2019 Set-2	2	1	4		
2020 Set-1	1	2	5		
2020 Set-2	2	2	6		
2021 Set-1	2	1	4		
2021 Set-2	3	1	5		
2022 Set-1	2	_	2		
2022 Set-2	2	1	4		
2023 Set-1	1	2	5		
2023Set-2	2	4	10		

## **Properties of Metals,** Stress & Strain

- The material that exhibits the same elastic properties in all direction at a point is said to be
  - (a) homogeneous
- (b) orthotropic
- (c) viscoelastic
- (d) isotropic [1995:1M]
- The stress-strain diagram for two materials A and B is shown below:



The following statements are made based on this diagram:

- I. Material A is more brittle than material B.
- II. The ultimate strength of material *B* is more than that of A.

With reference to the above statements, which of the following applies?

- (a) Both the statements are false
- (b) Both the statements are true
- (c) I is true but II is false
- (d) I is false but II is true

[2000: 1 M]

- The dimensions for the flexural rigidity of a beam element in mass (M), length (L) and time (T) is given by
  - (a)  $MT^{-2}$
- (b)  $ML^3T^{-2}$
- (c)  $ML^{-1}T^{-2}$
- (d)  $ML^{-1}T^2$  [2000 : 1 M]
- The shear modulus (G), modulus of elasticity (E) and the Poisson's ratio (v) of a material are related as

(a) 
$$G = \frac{E}{[2(1+v)]}$$
 (b)  $E = \frac{G}{[2(1+v)]}$ 

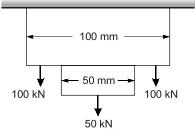
(b) 
$$E = \frac{G}{[2(1+v)]}$$

(c) 
$$G = \frac{E}{[2(1-v)]}$$
 (d)  $G = \frac{E}{[2(v-1)]}$ 

(d) 
$$G = \frac{E}{\left[2(v-1)\right]}$$

[2002:1 M]

A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self weight, the maximum tensile stress in N/mm<sup>2</sup> anywhere is



- (a) 16.0
- (b) 20.0
- (c) 25.0
- (d) 30.0

[2003 : 1 M]

- For linear elastic systeM, the type of displacement function for the strain energy is
  - (a) linear
- (b) quadratic
- (c) cubic
- (d) quartic [2004:1 M]
- The symmetry of stress tensor at a point in the body under equilibrium is obtained from
  - (a) conservation of mass
  - (b) force equilibrium equations
  - (c) moment equilibrium equations
  - (d) conservation of energy

[2005 : 1 M]

For an isotropic material, the relationship between the Young's modulus (E), shear modulus (G) and Poisson's ratio ( $\mu$ ) is given by

(a) 
$$G = \frac{E}{2(1+\mu)}$$
 (b)  $E = \frac{G}{2(1+\mu)}$ 

(b) 
$$E = \frac{G}{2(1 + \mu)}$$

(c) 
$$G = \frac{E}{(1+2\mu)}$$
 (d)  $G = \frac{E}{2(1-2\mu)}$ 

(d) 
$$G = \frac{E}{2(1-2\mu)}$$

[2007 : 1 M]

- A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10°C. If the coefficient of thermal expansion is  $12 \times 10^{-6}$  per °C and the Young's modulus is  $2 \times 10^5$  MPa, the stress in the bar is
  - (a) zero
- (b) 12 MPa
- (c) 24 MPa
- (d) 2400 MPa

[2007 : 2 M]

A rigid bar is suspended by three rods made of the same material. The area and length of the central rod are 3A and L, respectively while that of the two outer rods are 2A and 2L, respectively. If a downward force of 50 kN is applied to the rigid bar, the forces in the central and each of the outer rods will be

- (a) 16.67 kN each
- (b) 30 kN and 15 kN
- (c) 30 kN and 10 kN (d) 21.4 kN and 14.3 kN

[2007:2 M]

 $U_1$  and  $U_2$  are the strain energies stored in a prismatic bar due to axial tensile forces  $P_1$  and  $P_2$ , respectively. The strain energy U stored in the same bar due to combined action of  $P_1$  and  $P_2$ will be

- (a)  $U = U_1 + U_2$  (b)  $U = U_1 U_2$ (c)  $U < U_1 + U_2$  (d)  $U > U_1 + U_2$

[2007:2 M]

A mild steel specimen is under uniaxial tensile stress. Young's modulus and yield stress for mild steel are  $2 \times 10^5$  MPa and 250 MPa respectively. The maximum amount of strain energy per unit volume that can be stored in this specimen without permanent set is

- (a) 156 Nmm/mm<sup>3</sup>
- (b) 15.6 Nmm/mm<sup>3</sup>
- (c) 1.56 Nmm/mm<sup>3</sup>
- (d) 0.156 Nmm/mm<sup>3</sup>

[2008:1 M]

- A vertical rod PQ of length L is fixed at its top end P and has a flange fixed to the bottom end Q. A weight W is dropped vertically from a height h(< L)on to the flange. The axial stress in the rod can be reduced by
  - (a) increasing the length of the rod
  - (b) decreasing the length of the rod
  - (c) decreasing the area of cross-section of the rod
  - (d) increasing the modulus of elasticity of the material [2008 : 2 M]

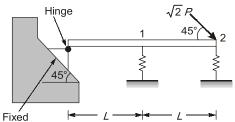
The number of independent elastic constants for a linear elastic isotropic and homo-geneous material is

- (a) 4
- (b) 3
- (c) 2
- (d) 1

[2010:1 M]

#### Linked Answer Questions 1.15 and 1.16:

A rigid beam is hinged at one end and supported on linear elastic springs (both having a stiffness of 'k') at points '1' and '2', and an inclined load acts at '2', as shown.



1.15 Which of the following options represents the deflections  $\delta_1$  and  $\delta_2$  at points '1' and '2'?

(a) 
$$\delta_1 = \frac{2}{5} \left( \frac{2P}{k} \right)$$
 and  $\delta_2 = \frac{4}{5} \left( \frac{2P}{k} \right)$ 

(b) 
$$\delta_1 = \frac{2}{5} \left( \frac{P}{k} \right)$$
 and  $\delta_2 = \frac{4}{5} \left( \frac{P}{k} \right)$ 

(c) 
$$\delta_1 = \frac{2}{5} \left( \frac{P}{\sqrt{2} k} \right)$$
 and  $\delta_2 = \frac{4}{5} \left( \frac{P}{\sqrt{2} k} \right)$ 

(d) 
$$\delta_1 = \frac{2}{5} \left( \frac{\sqrt{2}P}{k} \right)$$
 and  $\delta_2 = \frac{4}{5} \left( \frac{\sqrt{2}P}{k} \right)$ 

[2011:2 M]

If the load Pequals 100 kN, which of the following options represents forces  $R_1$  and  $R_2$  in the springs at points '1' and '2'?

- (a)  $R_1 = 20 \text{ kN} \text{ and } R_2 = 40 \text{ kN}$
- (b)  $R_1 = 50 \text{ kN} \text{ and } R_2 = 50 \text{ kN}$
- (c)  $R_1 = 30 \text{ kN} \text{ and } R_2 = 60 \text{ kN}$
- (d)  $R_1 = 40 \text{ kN} \text{ and } R_2 = 80 \text{ kN}$ [2011 : 2 M]

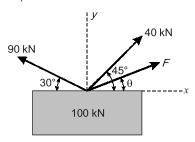
The Poisson's ratio is defined as

- axial stress lateral stress
- |lateral strain| axial strain
- |lateral stress axial stress
- (d) axial strain lateral strain

[2012 : 1 M]

- Creep strains are
  - (a) caused due to dead load only
  - (b) caused due to live load only
  - (c) caused due to cyclic load only
  - (d) independent of load [2013 : 1 M]

A box of weight 100 kN shown in the figure is to be lifted without swinging. If all forces are coplanar, the magnitude and direction  $(\theta)$  of the force (F)with respect to x-axis should be

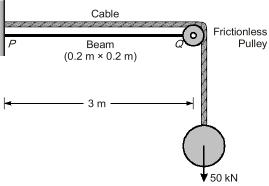


- (a)  $F = 56.389 \text{ kN} \text{ and } \theta = 28.28^{\circ}$
- (b)  $F = -56.389 \text{ kN} \text{ and } \theta = -28.28^{\circ}$
- (c)  $F = 9.055 \text{ kN and } \theta = 1.414^{\circ}$
- (d)  $F = -9.055 \text{ kN} \text{ and } \theta = -1.414^{\circ}$

[2014: 2 M, Set-I]

**MADE EASY** | Solid Mechanics

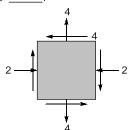
The values of axial stress ( $\sigma$ ) in kN/m<sup>2</sup>, bending moment (M) in kNm, and shear force (V) in kN acting at point P for the arrangement shown in the figure are respectively



- (a) 1000, 75 and 25 (b) 1250, 150 and 50
- (c) 1500, 225 and 75 (d) 1750, 300 and 100

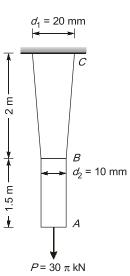
[2014 : 2 M, Set-II]

For the state of stresses (in MPa) shown in the figure below, the maximum shear stress (in MPa) is



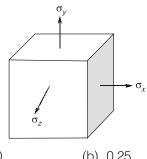
[2014: 2 M, Set-II]

A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity,  $E = 2 \times 10^5 \text{ MPa. When}$ subjected to a load  $P = 30\pi$  kN, the deflection at point A is \_\_\_\_ mm.



[2015: 2 M, Set-I]

An elastic isotropic body is in a hydrostatic state of stress as shown in the figure. For no change in the volume to occur, what should be its Poisson's ratio?



- (a) 0.00
- (b) 0.25
- (c) 0.50
- (d) 1.00

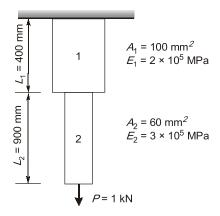
[2016: 2 M, Set-II]

An elastic bar of length L, uniform cross-sectional area A, coefficient of thermal expansion  $\alpha$ , and Young's modulus *E* is fixed at the two ends. The temperature of the bar is increased by T, resulting in an axial stress  $\sigma$ . Keeping all other parameters unchanged, if the length of the bar is doubled, the axial stress would be

- (a) σ
- (b)  $2\sigma$
- (c)  $0.5 \, \sigma$
- (d)  $0.25 \, \sigma$

[2017:1 M, Set-I]

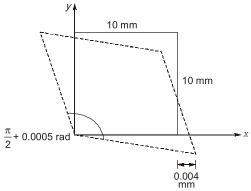
Consider the stepped bar made with a linear elastic material and subjected to an axial load of 1 kN, as shown in the figure.



Segments 1 and 2 have cross-sectional area of 100 mm<sup>2</sup> and 60 mm<sup>2</sup>. Young's modulus of  $2 \times 10^5$  MPa and  $3 \times 10^5$  MPa, and length of 400 mm and 900 mm, respectively. The strain energy (in N-mm upto one decimal place) in the bar due to the axial load is \_

[2017 : 2 M, Set-I]

In a material under a state of plane strain, a 10 × 10 mm square centered at a point gets deformed as shown in the figure.

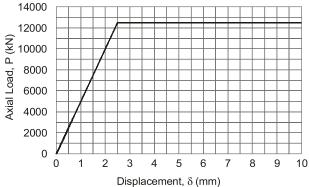


If the shear strain  $\gamma_{xy}$  at this point is expressed as 0.001 k (in rad), the value of k is

- (a) 0.50
- (b) 0.25
- (c) -0.25
- (d) -0.50

[2017: 1 M, Set-II]

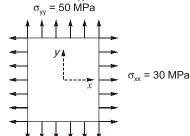
A 2 m long, axially loaded mild steel rod of 8 mm diameter exhibits the load-displacement (P- $\delta$ ) behavior as shown in the figure.



Assume the yield stress of steel as 250 MPa. The complementary strain energy (in N-mm) stored in the bar up to its linear elastic behaviour will be

[2017 : 2 M, Set-II]

A plate in equilibrium is subjected to uniform stresses along its edges with magnitude  $\sigma_{xx}$  = 30 MPa and  $\sigma_{yy}$  = 50 MPa as shown in the figure



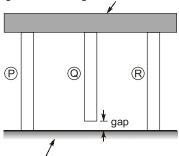
The Young's modulus of the material is  $2\times 10^{11}$  N/m² and the Poisson's ratio is 0.3. If  $\sigma_{zz}$  is negligibly small and assumed to be zero, then the strain  $\epsilon_{zz}$  is

- (a)  $-120 \times 10^{-6}$
- (b)  $-60 \times 10^{-6}$
- (c) 0.0
- (d)  $120 \times 10^{-6}$

[2018: 2 M, Set-I]

A rigid, uniform, weightless, horizontal bar is connected to three vertical members P, Q and R as shown in the figure (not drawn to the scale). All three members have identical axial stiffness of 10 kN/mm. The lower ends of bars P and R rest on a rigid horizontal surface. When NO load is applied, a gap of 2 mm exists between the lower end of the bar Q and the rigid horizontal surface. When a vertical load W is placed on the horizontal bar in the downward direction, the bar still remains horizontal and gets displaced by 5 mm in the vertically downward direction.

Rigid Uniform Weightless Horizontal Bar

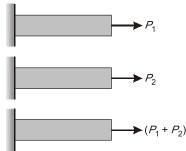


Rigid Horizontal Surface

The magnitude of the load W (in kN, round off to the nearest integer), is \_\_\_\_\_.

[2020 : 2 M, Set-I]

A prismatic linearly elastic bar of length, L, cross-sectional area A, and made up of a material with Young's modulus E, is subjected to axial tensile force as shown in the figures. When the bar is subjected to axial tensile force  $P_1$  and  $P_2$ , the strain energies stored in the bar are  $U_1$  and  $U_2$ , respectively.

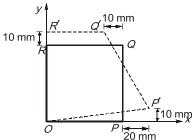


If U is the strain energy stored in the same bar when subjected to an axial tensile force  $(P_1 + P_2)$ , the correct relationship is

- (a)  $U = U_1 U_2$
- (b)  $U = U_1 + U_2$
- (c)  $U < U_1 + U_2$
- (d)  $U > U_1 + U_2$

[2020 : 2 M, Set-II]

31 A square plate O-P-Q-R of a linear elastic material with sides 1.0 m is loaded in a state of plane stress. Under a given stress condition, the plate deforM to a new configuration O-P'-Q'-R' as shown in the figure (not to scale). Under the given deformation, the edges of the plate remain straight.



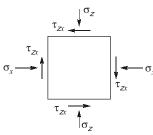
The horizontal displacement of the point (0.5 m, 0.5 m) in the plate O-P-Q-R (in mm, round off to one decimal place) is \_\_

[2021 : 2 M, Set-I]

- Strain hardening of structural steel means
  - (a) strengthening steel member externally for reducing strain experienced
  - (b) experiencing higher stress than yield stress with increased deformation
  - (c) strain occurring before plastic flow of steel material
  - (d) decrease in the stress experienced with increasing strain

[2021: 1 M, Set-II]

Stresses acting on an infinitesimal soil element are shown in the figure (with  $\sigma_z > \sigma_v$ ). The major and minor principal stresses are  $\sigma_1$  and  $\sigma_3$ , respectively. Considering the compressive stresses as positive, which one of the following expressions correctly represents the angle between the major principal stress plane and the horizontal plane?



- (a)  $\tan^{-1} \left( \frac{\tau_{zx}}{\sigma_1 + \sigma_3} \right)$  (b)  $\tan^{-1} \left( \frac{\tau_{zx}}{\sigma_3 \sigma_x} \right)$
- (c)  $\tan^{-1}\left(\frac{\tau_{zx}}{\sigma_1 \sigma_x}\right)$  (d)  $\tan^{-1}\left(\frac{\tau_{zx}}{\sigma_1 + \sigma_x}\right)$

[2022: 1 M, Set-III

For a linear elastic and isotropic material, the correct relationship among Young's modulus of elasticity (E), Poisson's ratio (v), and shear modulus(G) is

(a) 
$$G = \frac{E}{2(1+v)}$$
 (b)  $G = \frac{E}{(1+2v)}$   
(c)  $E = \frac{G}{2(1+v)}$  (d)  $E = \frac{G}{(1+2v)}$ 

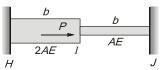
$$(b) \quad G = \frac{E}{(1+2v)}$$

(c) 
$$E = \frac{G}{2(1+v)}$$

(d) 
$$E = \frac{G}{(1+2v)}$$

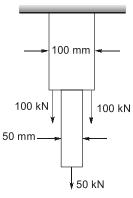
[2022 : 2 M, Set-II]

Consider two linearly elastic rods HI and IJ. each of length b, as shown in the figure. The rods are co-linear, and confined between two fixed supports at if and J. Both the rods are initially stress free, The coefficient of linear thermal expansion is a for both the rods. The temperature of the rod IJ is raised by  $\Delta T$ . whereas the temperature of rod HI remains unchanged. An external horizontal force P is now applied at node I. It is given that  $a = 10^{-}$  $^{6} \, ^{\circ}\text{C}^{-1}$ ,  $\Delta T = 50^{\circ} \, \text{C}$ , b = 2m, AE =  $10^{6} \, \text{N}$ . The axial rigidities of the rods HI and U are 2 AE and AE, respectively. To make the axial force in rod HI equal to zero, the value of the external force P (in N) is \_\_\_\_\_. (rounded off to the nearest integer).



[2022: 1 M, Set-II]

A hanger is made of two bars of different sizes. Each bar has a square cross-section. The hanger is loaded by three-point loads in the mid vertical plane as shown in the figure. Ignore the self-weight of the hanger. What is the maximum tensile stress in N/mm<sup>2</sup> anywhere in the hanger without considering stress concentration effects?



- (a) 15.0
- (b) 25.0
- (c) 35.0
- (d) 45.0

[2023: 1 M, Set-I]

A 2D thin plate with modulus of elasticity, E =1.0 N/m<sup>2</sup>, and Poisson's ratio,  $\mu$  = 0.5, is in plane stress condition. The displacement field in the plate is given by  $u = Cx^2y$ , and v = 0, where u and v are displacements (in m) along the X and Y directions, respectively, and C is a constant (in  $m^{-2}$ ). The distances x and y along

X and Y, respectively, are in m. The stress in the X direction is  $\sigma_{xx} = 40xy \text{ N/m}^2$ , and the shear stress is  $\tau_{XY} = \alpha x^2 N/m^2$ . What is the value of  $\alpha$ (in N/m<sup>4</sup>, in integer)?

[2023: 2 M, Set-II]

#### **Properties of Metals, Stress & Strain**

1.1	(d)	1.2	(c)	1.3	(b)	1.4	(a)	1.5	(c)	1.6	(b)	1.7	(c)	1.8	(a)	1.9	(c)
1.10	(c)	1.11	(d)	1.12	(d)	1.13	(a)	1.14	(c)	1.15	(b)	1.16	(d)	1.17	(b)	1.18	(a)
1.19	(a)	1.20	(b)	1.21	(5.0)	1.22	(15)	1.23	(c)	1.24	(a)	1.25	(35)	1.26	(d)		
1.27	(1562	25000)	1.28	(a)	1.29	(130)	1.30	(d)	1.31	(2.5)	1.32	2	(b)	1.33	(c)	1.34	(a)
1.35	(50)	1.36	(b)	1.37	(5)												

#### **Properties of Metals, Stress & Strain**

#### (d)

The material that exhibits the same elastic properties in all directions at a point is said to be isotropic.

If the material exhibits different properties in different directions, then it is said to be Anisotropic.

#### (c)

Since strain in material B is more, hence it is more ductile than material A i.e., material A is more brittle than material B. Hence statement I is true. Material A can reach upto higher stress level hence ultimate strength of material A is more than that of material B. Hence statement II is false.

Flexural regidity =  $EI = ML^{-1} T^{-2} \times L^4 = ML^3 T^{-2}$ 

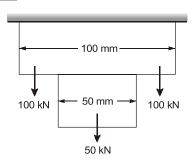
The stress in lower bar

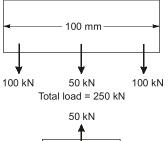
$$= \frac{50 \times 1000}{50 \times 50} = 20 \text{ N/mm}^2$$

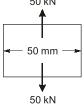
The stress in upper bar

$$= \frac{250 \times 1000}{100 \times 100} = 25 \text{ N/mm}^2$$

Thus the maximum tensile stress any where in the bar is 25 N/mm<sup>2</sup>.







Stress in upper bar

$$= \frac{250 \times 1000}{100 \times 100} = 25 \text{ N/mm}^2$$

Stress in lower bar = 
$$\frac{50 \times 10^3}{50 \times 50}$$
 = 20 N/mm<sup>2</sup>

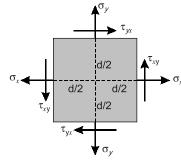
So maximum =  $25 \text{ N/mm}^2$ 

#### (b)

Strain Energy =  $\frac{1}{2} \times \sigma \times \varepsilon = \frac{1}{2} E \varepsilon^2$ 

Since strain is directly proportional to displacement so strain energy is directly proportional to quadratic equation of displacement.

#### (c)



Taking moment equilibrium about the centre, we get,

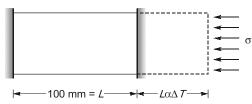
$$\tau_{yx} \times \frac{d}{2} + \tau_{yx} \times \frac{d}{2} = \tau_{xy} \times \frac{d}{2} + \tau_{xy} \times \frac{d}{2}$$

$$\vdots \qquad \qquad \tau_{xy} = \tau_{yx}$$

### (c)

Temperature stress

= 
$$\alpha TE$$
  
=  $12 \times 10^{-6} \times 10 \times 2 \times 10^{5}$   
=  $24 \text{ MPa}$ 



Due to temperature,

$$\Delta L = L\alpha\Delta T$$

But since support in fixed so, expansion is not allowed so stress is developed in the bar which is compressive in nature.

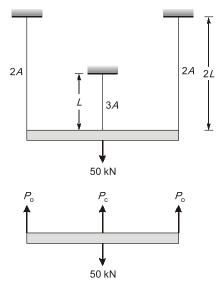
Expansion due Compression Now, to temperature = due to stress

$$L\alpha\Delta T = \frac{\sigma}{E} \times L$$

$$\sigma = E\alpha\Delta T = 1 \times 10^{5} \times 12 \times 10^{-6} \times 10$$

$$= 24 \text{ MPa}$$

#### .10 (c)



From symmetry, the force in each of outer rods is  $P_0$  and force in the central rod is  $P_C$ , then,

$$2P_0 + P_C = 50$$
 ...(i)

Also, the elongation of central rod and outer rods is same.

Since all the rods are connected by rigid bar. So, deflection in each rod will be same

$$\therefore \qquad \frac{P_0 L_0}{A_0 E} = \frac{P_C L_C}{A_C E}$$

$$\Rightarrow \qquad \frac{P_0 \times 2L}{2A} = \frac{P_C \times L}{3A}$$

$$\Rightarrow \qquad P_C = 3P_0 \qquad ...(ii)$$
Solving (i) and (ii), we get,
$$P_C = 30 \text{ kN and } P_0 = 10 \text{ kN}$$

#### (d)

Strain energy stored in a prismatic bar due to axial load  $P_1$ ,

$$U_1 = \frac{P_1^2 L}{2\Delta F}$$

Strain energy stored in a prismatic bar due to axial load,  $P_2$ ,

$$U_2 = \frac{P_2^2 L}{2AF}$$

Strain energy stored in a prismatic bar due to combined action of  $P_1$  and  $P_2$ ,

$$U = \frac{(P_1 + P_2)^2 L}{2AE}$$

$$U = \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{P_1 P_2 L}{AE}$$

$$U = U_1 + U_2 + \frac{P_1 P_2 L}{AE}$$
 
$$\Rightarrow \qquad U > U_1 + U_2$$

#### I.12 (d)

The strain energy per unit volume may be given as

$$U = \frac{1}{2} \times \text{Stresses} \times \text{Strain}$$

$$U = \frac{1}{2} \times \frac{\sigma_y^2}{E} = \frac{1}{2} \times \frac{(250)^2}{2 \times 10^5} = 0.156 \text{ N-mm/mm}^3$$

#### 1.13 (a)

The kinetic energy of the weight (W) at flange level will be transformed in the form of strain energy in the rod.

We know that.

 $U = \sigma^2/2E \times \text{Area of cross-section of rod} \times \text{length}$ 

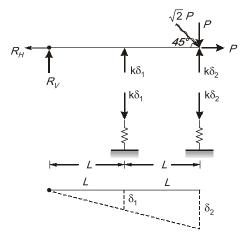
Since, energy remains constant, hence to reduce the axial stress in the rod, the length of the rod should be increased or area of cross-section of the rod should be increased or modulus of elasticity should be decreased.

#### 1.14 (c)

Type of material	Number of independent elastic constants
Isotropic	2
Orthotropic	9
Antisotropic	21

#### 1.15 (b)

The free diagram of the beam is shown below,



From similar triangles, we get,

$$\frac{L}{\delta_1} = \frac{2L}{\delta_2}$$

$$\Rightarrow \qquad \delta_2 = 2\delta_1 \qquad \dots (i)$$

Taking moments about hinge, we get,

$$P \times 2L - k\delta_2 \times 2L - k\delta_1 \times L = 0$$

$$\Rightarrow 2P - k(2\delta_2 + \delta_1) = 0$$

$$\Rightarrow 2P - k(4\delta_1 + \delta_1) = 0$$

$$\Rightarrow \delta_1 = \frac{2P}{5k}$$

From (i), we get,

$$\delta_2 = 2 \times \frac{2P}{5k} = \frac{4P}{5k}$$

#### 1.16 (d)

$$R_1 = k\delta_1 = k \times \frac{2P}{5k} = \frac{2 \times 100}{5} = 40 \text{ kN}$$
  
 $R_2 = k\delta_2 = k \times \frac{4P}{5k} = \frac{4 \times 100}{5} = 80 \text{ kN}$ 

#### 1.17 (b)

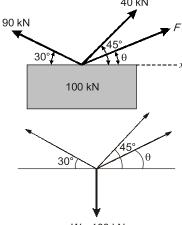
$$\mu = \left| \frac{\text{Lateral strain}}{\text{Axial strain}} \right|$$

#### 1.18 (a)

**Creep strains** occur with time at a constant level of stress. So they occur due to permanent load i.e., Dead Load.

#### 1.19 (a)

or



W= 100 kN

For no swinging force should be balanced i.e.,

$$\Sigma F_{H} = 0$$
 $\Sigma F_{V} = 0$ 
 $\Sigma F_{\text{horizontal}} = 0$ 
 $90 \cos 30^{\circ} = 40 \cos 45^{\circ} + F \cos \theta$ 
 $77.94 = 28.28 + F \cos \theta$ 
 $F \cos \theta = 49.66 \text{ kN}$  ...(i)

$$\Sigma F_{\text{vertical}} = 100$$

 $90 \sin 30^{\circ} + 40 \sin 45^{\circ} + F \sin \theta = W$ 

$$45 + 28.28 + F \sin \theta = 100$$

or 
$$F \sin \theta = 26.72 \,\text{kN}$$
 ...(ii)

Dividing eq. (ii) by eq. (i), we get

$$\tan \theta = \frac{26.72}{49.66} = 0.5380$$

$$\theta = \tan^{-1}(0.5380) = 28.28^{\circ}$$

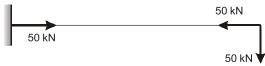
 $\theta = 28.28$  in eq. (i), we get Substituting

$$F\cos 28.28^{\circ} = 49.66$$

or 
$$F = \frac{49.66}{\cos 28.28^{\circ}} = \frac{49.66}{0.88} = 56.43 \text{ kN}$$

#### (b)

Loading after removing the cable,



Axial stress = 
$$\frac{50}{0.2 \times 0.2}$$
 = 1250 kN/m<sup>2</sup>

Bending moment =  $50 \times 3 = 150 \text{ kNm}$ Shear force = 50 kN

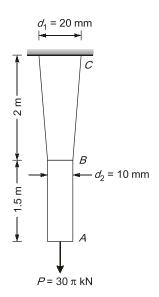
#### Sol.

$$\tau_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 5.0 \text{ MPa}$$

**NOTE**: In this case  $\sigma_{P1} = +ve$ 

So it can be found out by Mohr circle, otherwise always check for absolute shear stress.

#### Sol.



Total elongation,

AB is uniform

So, 
$$\Delta = \frac{PL}{AE}$$

BC is tapered

$$\Delta = \frac{PL}{\frac{\pi}{4} d_1 d_2 E}$$

$$\Delta = \Delta_{AB} + \Delta_{BC}$$

$$= \frac{PL}{AE} + \frac{4PL}{\pi d_1 d_2 E}$$

$$=\frac{30\pi\times10^3\times1.5\times10^3}{\frac{\pi}{4}\times(10)^2\times10^5}$$

$$+\frac{30\pi \times 10^{3} \times 2 \times 10^{3}}{\frac{\pi}{4} \times 10 \times 20 \times 2 \times 10^{5}}$$

$$=(9+6) \text{ mm} = 15 \text{ mm}$$

#### (c)

Volumetric strain,

$$\in_{V} = \left(\frac{\sigma_{x} + \sigma_{y} + \sigma_{z}}{3}\right) (1 - 2\mu)$$

$$\Rightarrow \frac{\delta V}{V} = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{3}\right) (1 - 2\mu)$$

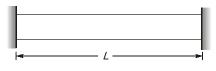
$$\Rightarrow \text{ Either } \sigma_x + \sigma_y + \sigma_z = 0 \text{ or } 1 - 2\mu = 0$$

$$\Rightarrow 1 - 2\mu = 0$$

$$\Rightarrow 1-2\mu=0$$

$$\mu = 0.5$$

#### (a)



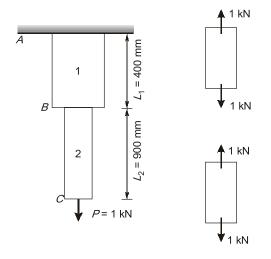
$$\sigma = \alpha T E$$

- :. Length have no effect on thermal stress.
- $\therefore$  Axial stress is only ' $\sigma$ '.

#### Sol.

$$A_1 = 100 \text{ mm}^2, EI = 2 \times 10^5 \text{ MPa}$$

$$A_2 = 60 \text{ mm}^2$$
,  $EI = 3 \times 10^5 \text{ MPa}$ 



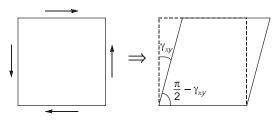
$$\Delta_{AC} = \Delta_{AB} + \Delta_{BC}$$

$$= \frac{1 \times 10^{3} \times 400}{100 \times 2 \times 10^{5}} + \frac{1 \times 10^{3} \times 900}{60 \times 3 \times 10^{5}}$$

$$= 0.02 + 0.05 = 0.07 \text{ mm}$$

$$U = \frac{1}{2} \times P \times \Delta = \frac{1}{2} \times 1 \times 1000 \times 0.07 = 35 \text{ N-mm}$$

According to the sign convention,

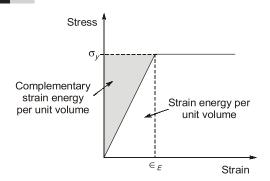


+ve shear stress

+ve shear strain

In question since angle has been increase therefore shear strain should be negative.

#### Sol.



Elastic strain, 
$$\epsilon_E = \frac{\Delta L}{L} = \frac{2.5}{2000} = 1.25 \times 10^{-2}$$

∴Elastic strain energy = 
$$\frac{1}{2}\sigma_y \in_E AL$$

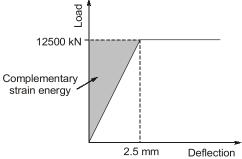
$$= \frac{1}{2} \times 250 \times 1.25 \times 10^{-3} \times \frac{\pi}{4} \times 8^2 \times 2000$$

= 15707.96 Nmm

Note: For linear elastic material both complementary energy and strain energy is same.

#### OR

By considering given graph in question, between Axial Load and Displacement the solution will be as follows:



Complementary stain energy,

$$U = \frac{1}{2}P\delta = \frac{1}{2}(12500 \times 10^3) \times 2.5$$
  
= 15625000 Nmm

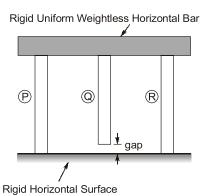
It means there seeM some error in the given data.

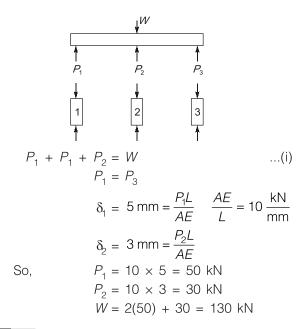
$$\sigma_{xx} = 30 \text{ MPa}, \ \sigma_{yy} = 50 \text{ MPa}, \ \sigma_{zz} = 0$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \mu \frac{\sigma_{xx}}{E} - \mu \frac{\sigma_{yy}}{E} = -\frac{\mu}{E} (\sigma_{xx} + \sigma_{yy})$$

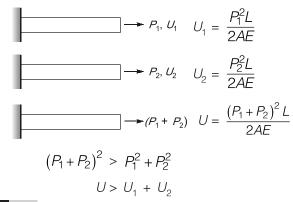
$$= -\frac{0.3}{2 \times 10^5} (30 + 50) = -120 \times 10^{-6}$$

#### Sol.

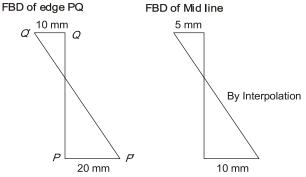




#### 1.30 (d)



#### 1.31 Sol.



So horizontal displacement of the point (0.5 m, 0.5 m)

$$= -2.5 \text{ mm} + 5 \text{ mm} = 2.5 \text{ mm}$$

#### 1.33 (c)

$$\begin{split} & \Sigma F_x = 0 \\ \Rightarrow & -\sigma_x(BC) - \tau_Z \times (AB) + \sigma_1 \sin\theta = 0 \end{split}$$

$$\Rightarrow \sigma_{x} \left( \frac{AC \sin \theta}{\cos \theta} \right) + \tau_{zx} \left( \frac{AC \cos \theta}{\cos \theta} \right)$$

$$= \sigma_{1} \frac{\sin \theta \times AC}{\cos \theta}$$

$$\Rightarrow \sigma_{x} \tan \theta + \tau_{zx} = \sigma_{1} \tan \theta$$

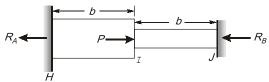
$$\Rightarrow \tan \theta \ (\sigma_{1} - \sigma_{x}) = \tau_{zx}$$

$$\Rightarrow \tan \theta = \left( \frac{\tau_{zx}}{\sigma_{1} - \sigma_{x}} \right)$$

#### 1.34 (a)

 $E = 2G(1 + \mu);$  G = Shear modulus $\mu = \text{Poission's ratio};$  E = Young's modulus

#### 1.35 Sol.



For horizontal force equilibrium,

$$R_A + R_B = P$$

Since there is no axial force in rod HI, thereore  $R_A = 0$ .

Now check for rod IJ,

$$\therefore$$
  $R_R = P$ 

Now, as rod IJ is fixed from both end, so net deflection due to increase in temperature will be 0.

$$\therefore b\alpha T + \left[ -\frac{Pb}{AE} \right] = 0$$

$$\Rightarrow \qquad b\alpha T = \frac{Pb}{AE}$$

$$\Rightarrow \qquad P = AE\alpha T$$

$$\Rightarrow \qquad P = 10^{6} \times 10^{-6} \times 50$$

$$\Rightarrow \qquad P = 50 \text{ N}$$

#### .36 (b)

